

# Study on Partial Returns to Competing Retailers under Uncertain Demand

Sun Ting

School of Management and Economics, University of Electronic Science and Technology, Chengdu,  
China P.R, 610054  
(E-mail: st2009@gmail.com)

**Abstract** This paper discusses the partial return policies in the channel of single manufacturer and two competing retailers under demand uncertainty. Substantial literature has been reviewed about full return policies. They ignore the partial return policies which are more general in supply chain. In contrast to no return and full return policies, the author investigates the difference in profit of the manufacturer and retailers with partial return policies. Under certain circumstance, the paper shows that the partial return policies can be a win-win strategy for all parties. Pareto improvement is verified to make the contract accepted by both manufacturer and retailer.

**Key words** Partial returns; Demand uncertainty; Pareto improvement; Competing retailers

## 1 Introduction

Manufacturers' returns policies are a common feature in the distribution of many products (e.g., news-papers, periodicals, records, etc.) The obvious rationale for returns policies is insurance. Practitioners, not surprisingly, have a different perspective and view returns as a cost of doing business. Manufacturer use various policies fitting this model, range from a full credit on all unsold goods to no credit for unsold items. Middle road strategies, such as full credit for partial returns or partial credit for full returns, are also currently in use. A returns policy eliminates the cost to retailers of excess inventory and so encourages retailers to stock aggressively when faced with uncertainty in demand.

In the general setting, when there are competing retailers and demand is uncertain, there is a trade-off for the manufacturer between the benefits (more intense retail competition) and the costs of a returns policy. The paper shows that several such policies currently in effect are suboptimal. These include those where the manufacturer offers retailers full credit for all unsold goods or where no returns of unsold goods are permitted. We show that neither a policy of allowing for unlimited returns at full credit nor one which allows for no returns could be optimal. Limiting returns to a fixed percentage of sales may allow an optimal policy to be developed; however, such a policy will not be optimal in a multi-retailer environment.

We demonstrate that an optimal policy in the multi-retailer environment is only achievable if unlimited returns are permitted for partial credit. In this case, formulae for the optimal price to be charged the retailer as well as the partial credit amount are presented along with formulae for the expected profit achievable by both the retailer and manufacturer. That is, while the manufacturer may set a coordinated pricing and return policy which increases the average retailer's expected profit, some retailers may actually experience a decrease in expected profit due to the policy change. As a consequence, some retailers may choose to no longer carry the product while, at the same time, others may now find it attractive to do so.

In this context, Padmanabhan V. and I. P. L (1995)<sup>[1]</sup> consider the impact of two factors-retail competition and demand uncertainty on a manufacturer's decision whether to accept returns. We show that a returns policy can benefit a manufacturer by inducing retailers to compete more intensely. Pasternack, B.A.<sup>[2]</sup> claimed that an appropriate return policy can fully coordinate a single-supplier single-retailer supply chain. Bernstein, F. (2005)<sup>[3]</sup> investigate the equilibrium behavior of decentralized supply chains with competing retailers under demand uncertainty.

Here we extend the work of Padmanabhan and Png(1997)<sup>[4]</sup> to include partial returns. The setting for our research is the distribution of products with uncertain demand, limited shelf lives, and retail competition. The market share, and therefore profits, of each member are based on the quantities brought to the market and the prices charged by these retailers. We focus on a linear demand model with a simple uncertain demand distribution that includes only two realizations Mills,E.S(1959)<sup>[5]</sup>. Our objective is to provide a better understanding of when manufacturers should adopt returns policies.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 considers the centralized system. Section 4 shows the influence of partial returns for all the parties by

numerical and graphic analysis. We give the operational and strategic decisions with competing retailers at uncertain demand. Section 5 concludes the paper and gives the future research.

## 2 Basic Setting

Notation

Subscript $i, j$	denote the two retailers in the supply chain
$p_{ix}, p_{jx}$	Retail price at demand state $x$
$s_i, s_j$	Production and order quantity
$c$	per unit production cost
$w$	wholesale price (when this chain is not Vertically integrated chain)
$q_{ix}, q_{jx}$	sales by the retailer at demand state $x$
$\pi_x^c$ and $\pi^c$	profit at demand state $x$ and expected profit, respectively
$\pi_m$	manufacturer's profit (when this chain is not Vertically integrated chain)
$\pi_{rix}, \pi_{rjx}$ and $\pi_{ri}, \pi_{rj}$	retailer's profit at demand state $x$ and expected profit, respectively
$d_{ix}$	market demand faced by the chain at demand state $x$
$u$	probability of low demand state, $1-u$ is the probability of high demand state
$q_i = \min(s_i, d_i)$	

To keep things tractable we assume that the demand is uncertain with just two possible realizations (Desheng Wu 2009)<sup>[6]</sup>: a low value,  $l$ , with probability  $u$ , and a high value,  $h$ , with probability  $1-u$ . Note that when  $u=1$  or  $u=0$  we obtain the certain demand case. We consider additive demand where the  $i^{th}$  retailer's demand depends on three elements: the primary uncertain demand at state  $x \in (h, l)$ , (we assume  $a_l < a_h$ ) its own price,  $p_{ix}$ , and the competitor's price  $p_{jx}$  ( $j \neq i$ ) through a substituting coefficient  $b \in (0, 1)$ :

$$\begin{aligned} d_{ix} &= a_x - p_{ix} + bp_{jx} \\ d_{jx} &= a_x - p_{jx} + bp_{ix} \end{aligned} \tag{1}$$

As reflected in (1),  $b=0$  implies that the chains are independent of each other while  $b=1$  implies that the products are not differentiable. As before we assume that demand uncertainty is resolved before the retail price  $p_i$  is chosen.

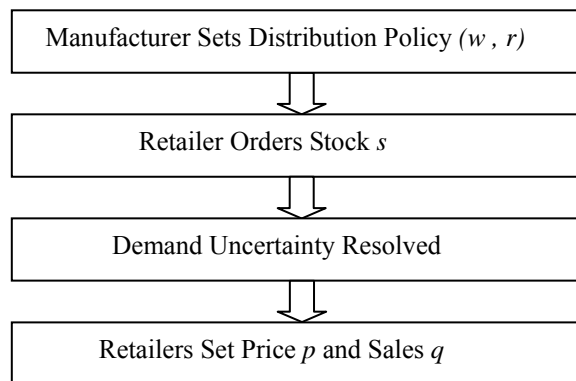


Figure 1 sequence of moves

Let us review the information structure and sequence of moves. Initially, all parties are uncertain about the primary demand. In the first stage, the manufacturer sets the distribution policy as a Stackelberg leader. The distribution policy includes a wholesale price,  $w$ , and possibly a buyback price  $r$ . In the second stage, the retailers order stock  $s$ , while, in the third stage, the retailers set price, decide the sales quantity.

In this model of supply chains, when demand uncertainty is resolved before the retail price decision is made, the retail price will be chosen such that: at the high demand state,  $s = q$ , and at the low

demand state when the per unit production cost is not high enough(Desheng Wu 2009)<sup>[6]</sup>, then in the low demand state sales are not constrained by the stock  $s$ .

### 3 Vertically Integrated Supply Chain

Vertically integrated chain is a chain with a single owner, the manufacturer and the retailer are two branches of the same company and thus both face the same demand. The company sequentially decides on production quantity  $s$  and retailer price  $p$ . When the demand is low, the company determines the optimal price by:

$$\text{Max}_{p_{il}, p_{jl}} \{\pi_i^c\} = \text{Max}_{p_{il}, p_{jl}} p_{il}q_{il} + p_{jl}q_{jl} - c(s_i + s_j) = (a_i - p_{il} + bp_{jl})p_{il} + (a_i - p_{jl} + bp_{il})p_{jl} - c(s_i + s_j) \quad (2)$$

FOC (First Order Conditions) with (2)

$$\frac{\partial \pi_i^c}{\partial p_{il}} = a_i - 2p_{il} + 2bp_{jl} = 0; \frac{\partial \pi_i^c}{\partial p_{jl}} = a_i + 2bp_{il} - 2p_{jl} = 0 \quad (3)$$

Solving the reaction functions leads to the optimal price and sales for the low demand state,;

$$p_{il}^c = p_{jl}^c = \frac{a_i}{2(1-b)} \quad q_{il}^c = q_{jl}^c = \frac{a_i}{2} \quad (4)$$

Thus, the company's optimal profit at the low demand state is:

$$\pi_i^c = p_{il}q_{il} + p_{jl}q_{jl} - c(s_i + s_j) \quad (5)$$

When demand is high, we follow our mention that retail price is chosen to clear inventory. Thus, The company determines optimal prices by solving the reaction functions that follow from:

$$p_{ih} = \frac{a_h + ba_h - s_i - bs_j}{b^2 - 1}; p_{jh} = \frac{a_h + ba_h - bs_i - s_j}{b^2 - 1}$$

Therefore, the Supply Chain's expected profit is:

$$\pi^c = u\pi_i^c + (1-u)\pi_h^c = \frac{ua_i^2}{2(1-b)} + (1-u)\left(\frac{(a_h + ba_h - bs_i - s_j)s_j}{b^2 - 1} + \frac{s_i(a_h + ba_h - s_i - bs_j)}{b^2 - 1}\right) - c(s_i + s_j) \quad (6)$$

FOC(6) with respect to  $s_i, s_j$  leads to the reaction functions whose solution gives the order quantity the optimal price (for both retailers) at high demand state of the Vertically integrated equilibrium

$$s_i = s_j = \frac{-c + bc + a_h - ua_h}{2(1-u)} \quad p_{ih} = p_{jh} = \frac{-c + bc - a_h + ua_h}{2(b-1)(1-u)} \quad (7)$$

Thus, The Vertically integrated supply chains expected profit is

$$\pi^c = \frac{(1-u)ua_i^2 + [(1-u)a_h - (1-b)c]^2}{2(1-b)(1-u)} \quad (8)$$

### 4 Partial Returns Policies

Next, we analyze a wholesale price case where the manufacturer sets and gives a partial refund for unsold stocks. We assert a similar result to the one used in Padmanabhan and Png(1997,2004) for Monopolistic markets. Namely, sales are determined by quantity ordered at the high demand state and by demand at the low demand state. Since the manufacturer accepts returns, if retailer orders stock  $s$ , but sells only  $q < s$  units, it can return the unsold  $(s - q)$  units and need pay only  $w\varphi(s - q)$  to the manufacturer. Thus, in the third stage, the retailer has no stocking constraint and prices to maximize profit. Solving the first order conditions for the retailers yields the retail prices and the retail sales (9),

$$\pi_{ril} = p_{il}q_{il} - ws_i + r(s_i - q_{il}) = (p_{il} - \varphi w)q_{il} - w(1 - \varphi)s_i = (p_{il} - \varphi w)(a_i - p_{il} + bp_{jl}) - w(1 - \varphi)s_i$$

$$\pi_{rjl} = p_{jl}q_{jl} - ws_j + r(s_j - q_{jl}) = (p_{jl} - \varphi w)q_{jl} - w(1 - \varphi)s_j = (p_{jl} - \varphi w)(a_i - p_{jl} + bp_{il}) - w(1 - \varphi)s_j$$

$$p_{il} = p_{jl} = \frac{w\varphi + a_i}{2-b} \quad q_{il} = q_{jl} = \frac{a_i - (1-b)w\varphi}{2-b} \quad (9)$$

In this case the retailer faces a production cost  $w$  rather than  $c$ . The retailer's expected profit is:

$$\begin{aligned} \pi_{ri} &= u[(p_{il} - \varphi w)q_{il} - w(1 - \varphi)s_i] + (1 - u)(p_{ih} - w)s_i \\ \pi_{rj} &= u[(p_{jl} - \varphi w)q_{jl} - w(1 - \varphi)s_j] + (1 - u)(p_{jh} - w)s_j \end{aligned} \tag{10}$$

Taking the FOC we have the reaction function whose solution is:

$$s_i = s_j = \frac{(1 + b)[w(1 - b)(1 - u\varphi) + (1 - u)a_h]}{(2 + b)(1 - u)} \tag{11}$$

Substituting (8) into the manufacturer's expected profit function and taking FOC with respect to  $w$ ;

$$\pi_m = 2\{u[ws_i - r(s_i - q_{il})] + (1 - u)ws_i - cs_i\} = 2[ruq_{il} - (c + ru - w)s_i] \tag{12}$$

Solving the manufacturer's reaction Function. Hence, Manufacturer's equilibrium wholesale price  $w$  is

$$w = \frac{(2 + b)(-1 + u)u\varphi a_l + (-2 - b + b^2)(-1 + u)(-1 + u\varphi)a_h - (2 - b - 2b^2 + b^3)(-1 + u\varphi)c}{2(-1 + b)\{-2 - 4u^2\varphi^2 + 2u\varphi(2 + \varphi) + b^2(-1 + u\varphi)^2 + b[-1 - 2u^2\varphi^2 + u\varphi(2 + \varphi)]\}} \tag{13}$$

#### 4.1 Numerical examples

In this section, we investigate the increase in channel profits resulting from partial returns. we choose  $c = 0, b = 1/2, a_l = 1$  to simplify the model, that is, the substituting coefficient is not very high, and per unit production cost is zero. Substituting them into(12),we have

$$\pi_m = \frac{(u - 1)[10u\varphi + (9 - 9u\varphi)a_h]^2}{15\{-9 + u\varphi[18 + (-10 + u)\varphi]\}} \tag{14}$$

##### 4.1.1 Low demand uncertainty

When demand uncertainty is pretty low, consider  $u > 1/2$ , let  $u = 9/10, a_h = 5$  we have the expected Profit of Manufacturer and Retailers(M&R) at low demand state.(Figure 2)

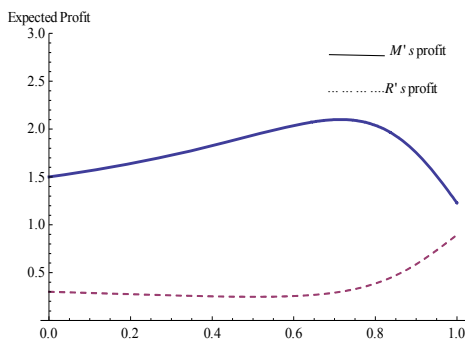


Figure 2 Profits of M&R at low demand state

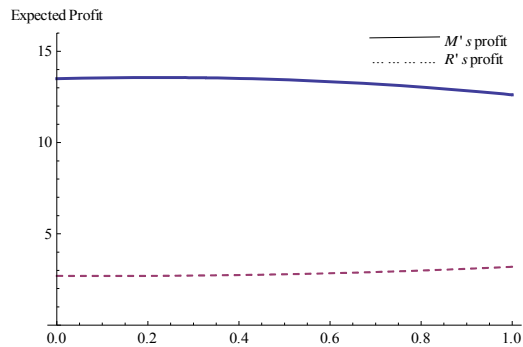


Figure 3 Profit of M&R at high demand state

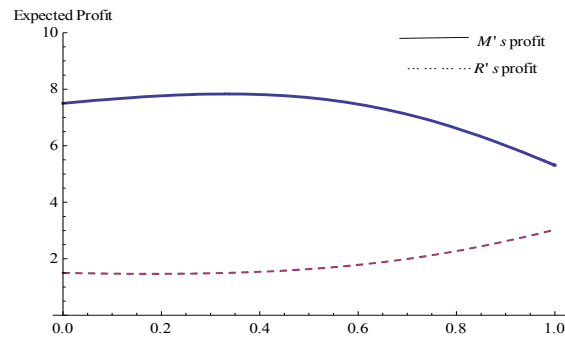
At low demand state, manufacturers expected profit with partial returns is obviously more than no returns and full returns. Retailers prefer the full return policies, however it can't be accepted by the manufacturer. When  $0 < \varphi < 5/7$  partial returns intensify the competition between the retailers, thus reduce their revenue. When manufacturer raise the value of  $\varphi$ , retailer will increase stock  $s$ , and benefit from the partial return policies.

Proposition 1 At low demand state, when  $5/7 < \varphi < 9/10$  all of the manufacturer and retailers' expected profit will more then the condition of  $\varphi=0$ , thus, the partial policies ( $5/7 < \varphi < 9/10$ ) adopted by the manufacturer is a win-win strategy. It is Pareto improvement for all the parties.

Proposition 2 At high demand state(Figure 2), with the increase of the value  $\varphi$ , manufacturers expected profit will reduce. Thus, manufacturer dislikes offering partial returns. Compared with low demand state, partial refund has little influence with the retailers at high demand state.

##### 4.1.2 High demand uncertainty

When demand uncertainty is extremely high, consider  $u = 1/2$ , uncertainty is maximized.



**Figure 4 Profit of M&R at high uncertainty state**

### Proposition 3

At high demand uncertainty state (Figure 4), the manufacturer is inclined to reduce the value of  $\varphi$ , thus, it can optimal the revenue at the high demand uncertainty circumstance. When  $0 < \varphi < 2/5$  all of the parties will benefit from the partial return policies. However, full return policies will definitely harm the manufacturer's expected profit.

## 5 Conclusions

We build a model of one manufacturer and two competing retailers to illustrate the impact of partial returns at different demand state. When demand uncertainty is pretty low, partial returns can be a Pareto improvement for all the parties at low demand state. Manufacturer adopts returns policies will stimulate the stock and sales of the retailers, thus improve the revenue of all parties. By contrast, there is no need to implement full returns at high demand state. When demand uncertainty is very high, the manufacturer would prefer setting a comparable low partial refund to avoid excess stock loss.

We have not considered risk-preference among retailers, as we assumed that all parties were risk-neutral. If the retailers are risk-averse, the wholesale price with returns should higher than in our setting. In addition, we ignored the costs of administering and implementing partial returns. These costs also should be considered when a manufacturer decides whether or not to accept returns. Besides, there are many interesting opportunities for future research in this area. Consideration of manufacturer competition and information asymmetry among the parties would add further richness to the model.

## References

- [1] Padmanabhan V. I. P. L. Png Returns Policies: Make Money by Making Good[J]. Sloan Management Review, 1995(Fall): 65-72
- [2] Pasternak, B.A. Optimal Pricing and Returns Policies for Perishable Commodities[J]. Marketing Science, 1985, 4(2):166-176
- [3] Bernstein, F. and Federgruen, A. Decentralized Supply Chains with Competing Retailers Under Demand Uncertainty[J]. Management Science, 2005, 51(1):18-29
- [4] Padmanabhan, V. and Png, I. P. L. Manufacturers Returns Policies and Retail Price Competition[J]. Marketing Science, 1997, 16(1):81-94
- [5] Mills, E.S. Uncertainty and Price Theory[J]. Quarterly Journal of Economics, 1959, (73):116-130
- [6] Desheng Wu, Opher Baron, Oded Berman, Bargaining in Competing Supply Chains with Uncertainty[J]. European Journal of Operational Research, 2009, (197):548-556